

# Long-term prognosis of discharge in selected rivers of the Danube and Elbe basins

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## 1 Introduction

Since the very inception of hydrology as a scientific discipline, hydrologists and climatologists around the world have been trying to make science-based prediction of future development in the hydrosphere. Generally it is expected that the increase of temperature will increase evapotranspiration in summer and decrease the runoff. Consecutively the oxygen regime and stream water quality will worsen.

There are two main approaches to stream runoff prognosis in the next decades:

1. Statistical analysis of long-term discharge series followed by prognosis using stochastic autoregressive models;
2. Application of hydrological rainfall-runoff models based on precipitation-temperature-stream runoff relations. The precipitation and temperature data are modified according to selected climate scenarios for future time horizons (most frequently 2025, 2050 or 2075).

We used the first method and identified the changes in statistical characteristics of the discharge using detailed statistical analysis of the daily, monthly and annual time series. The second part of this study is devoted to the long-term prediction of the monthly discharge of the Danube and Elbe Rivers by applying stochastic methods.

## 2 Description of the basins and data

The Danube and Elbe are among the largest rivers of Europe. The basin area upstream of the Bratislava, and the Wittenberge gauge is 131 338 km<sup>2</sup>, and 123 532 km<sup>2</sup>, respectively (Table 1, Fig.1). The Elbe Basin is one of the driest river basins in Europe according to the specific yield. The mean annual runoff of the Elbe River at Wittenberge is only one third of the Danube runoff at Bratislava (495 mm to 173 mm) (Table 1).

The increase in air temperature has not affected the annual mean discharge rate observed in the Upper Danube at Bratislava station during 1876–2005 (Fig.2), while the seasonal pattern has been significantly changed by increasing discharge rates in December–April and decreasing rates for June–August.

The mean annual discharge of the Elbe River is slightly increasing in 1875–2005. The increase is mainly due to increase of discharge in the winter period.

On the other side, in Slovak Danube tributaries Morava and Vah decreasing rates of the discharge were observed.

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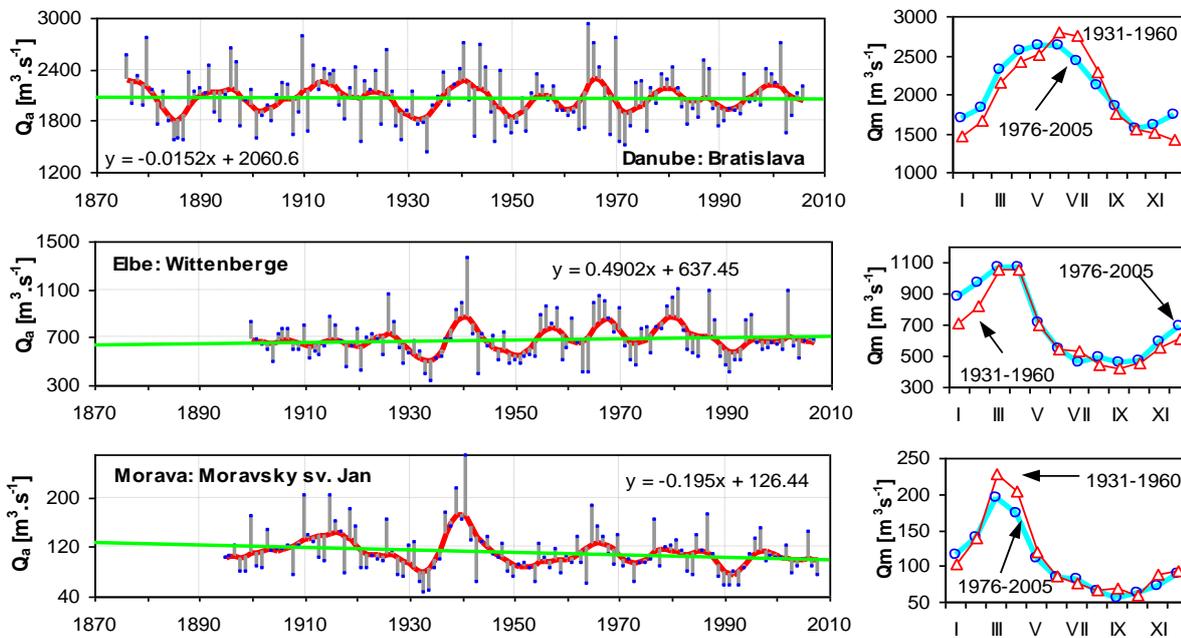
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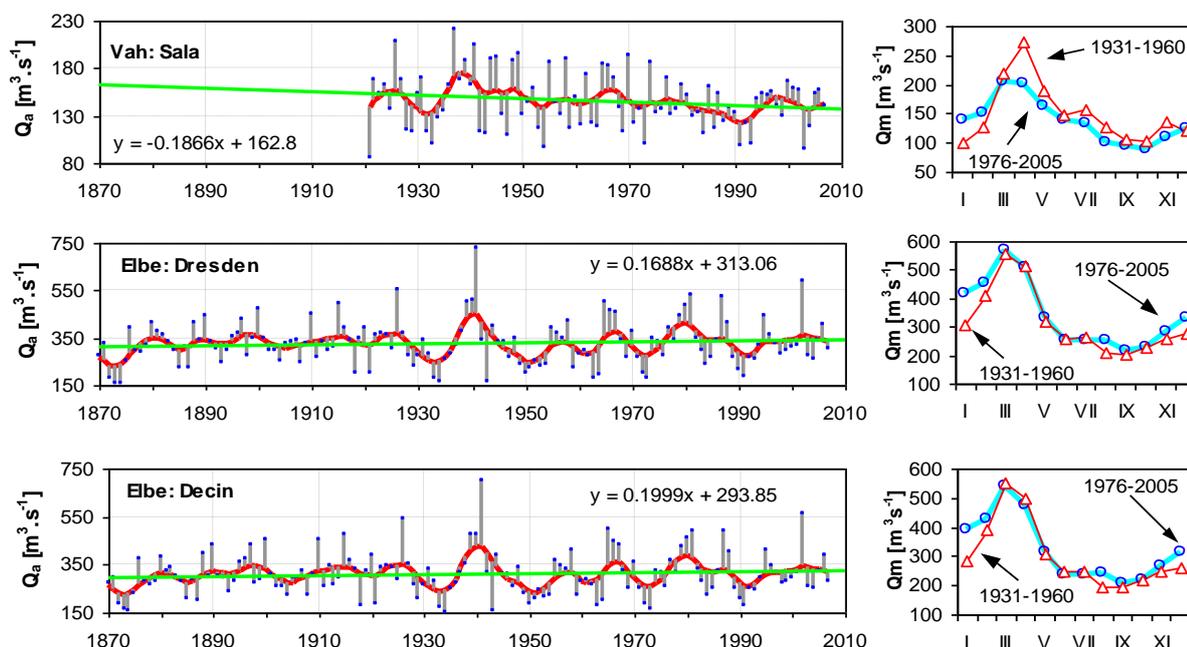


Figure 1. Scheme of the Elbe and the Upper Danube River Basins

Table 1. Long-term mean annual and daily discharge characteristics of the Danube at Bratislava and Labe at Wittenberge. A – basin area; Q/q/R – long-term average annual discharge/specific yield/runoff depth;  $Q_{d,min}/Q_{d,max}$  – minimal/maximal daily discharge [ $m^3 s^{-1}$ ];  $Q_{m,min}/Q_{m,max}$  – minimal/maximal monthly discharge [ $m^3 s^{-1}$ ]

	A [ $km^2$ ]	Q [ $m^3 s^{-1}$ ]	q [ $l \cdot s^{-1} km^{-2}$ ]	R [mm]	3	Q <sub>d,min</sub>	4	Q <sub>d,max</sub>	Q <sub>m,min</sub>	Q <sub>m,max</sub>
Danube: Bratislava	131338	2060	15.68	494.5	580	10810	633	7324		
Elbe: Wittenberge	123532	678	5.49	173.1	120	3690	132	2345		





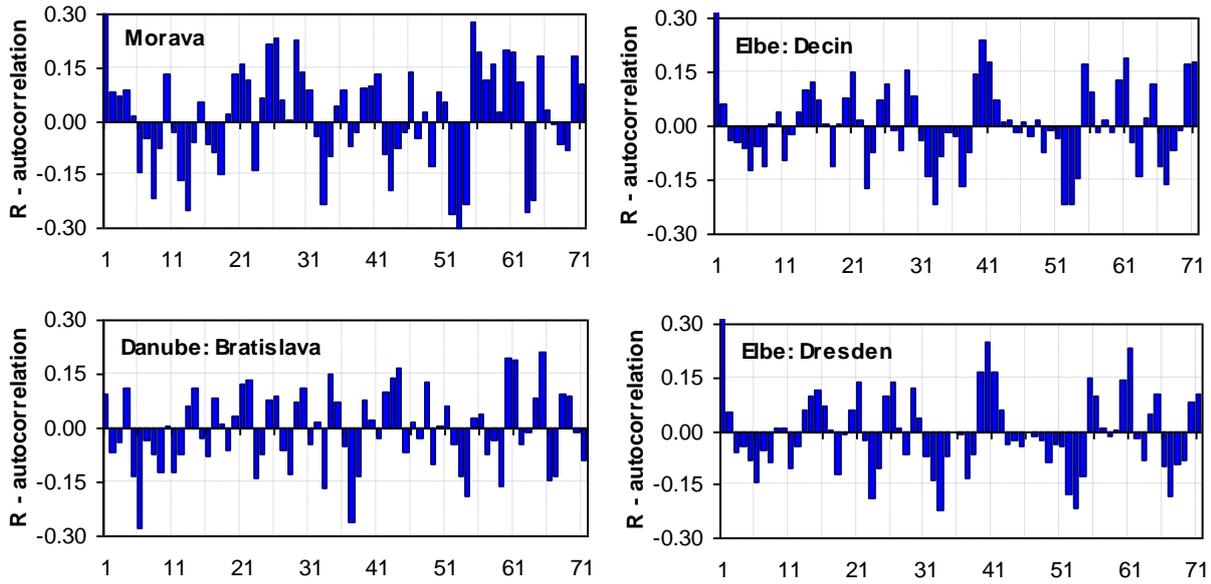
**Figure 2.** Mean annual discharge at selected stations of the Danube and Elbe Basins during the whole observation periods (left panel) and comparison of the mean monthly runoff regime in two different 30-year periods (right panel).

## 5 Multi-annual variability of runoff – Hurst phenomenon

More than 50 years ago, by the studies of long-term storage requirements on the Nile River, Hurst (1951) discovered a special behavior of the hydrological and other geophysical time series, which has become known as the “Hurst phenomenon”. This behavior is essentially the tendency of the wet years to cluster into wet periods, or of the dry years to cluster into drought periods (Lin & Lye, 1994).

The basic mathematical expression of this phenomenon can be written as:  $R_n/S_n = (n/2)^h$ ; where  $R_n$  and  $S_n$  are the sample-adjusted range of cumulative departures from the arithmetical sample mean and the sample standard deviation, respectively, for a given time series of length  $n$ . Coefficient  $h$  denotes the Hurst coefficient. The Hurst parameter is commonly used as an indicator of the time-series' long-range memory, i.e. persistence (Markovic & Koch, 2006). A value  $0 < h < 0.5$  corresponds to antipersistent,  $h = 0.5$  to white, and  $0.5 < h < 1$  to correlated or persistent noise. Moreover, a time-series of  $h = 1$  is called pink noise, of  $h = 1.5$  brown noise, and of  $h > 1.5$  black noise. For precipitation and river discharge time series  $0.5 < h < 1$  are most likely encountered. Hurst observed, that on average  $h = 0.73$ . The Hurst coefficient  $h$  of the average annual Danube discharge at Bratislava (1876–2005) is 0.59, of the Elbe River at Dresden 0.64.

In this part of the study, we focused on dry and wet multi-annual cycles identification of the annual runoff characteristics for the Danube River at Bratislava, Morava River at Moravsky sv. Jan, and Elbe River at Dresden and Decin. The multi-annual cyclic component of the average annual discharges was identified by auto-correlograms (Fig. 3) and spectral analysis. Figure 4 depicts combined periodograms (Pekárová, 2009) of average annual discharges. The spectral analysis confirmed that the occurrence of multi-annual cycles within dry and wet periods (in both basins) is of the following durations: 2.4; 3.6; 5-6; 7; 7.8; 10-11; 12-14; 20-22; and 28-30 years.

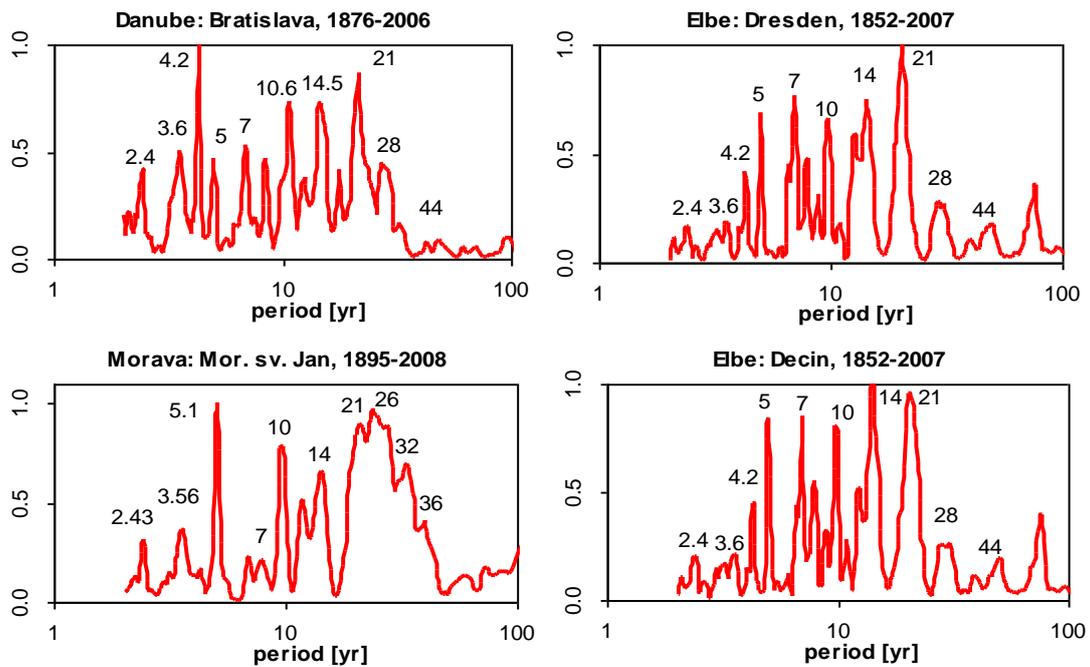


**Figure 3.** The auto-correlograms of the average annual discharges of the Morava River at M. Jan; the Danube River at Bratislava; the Elbe River at Decin and at Dresden (time lag in years).

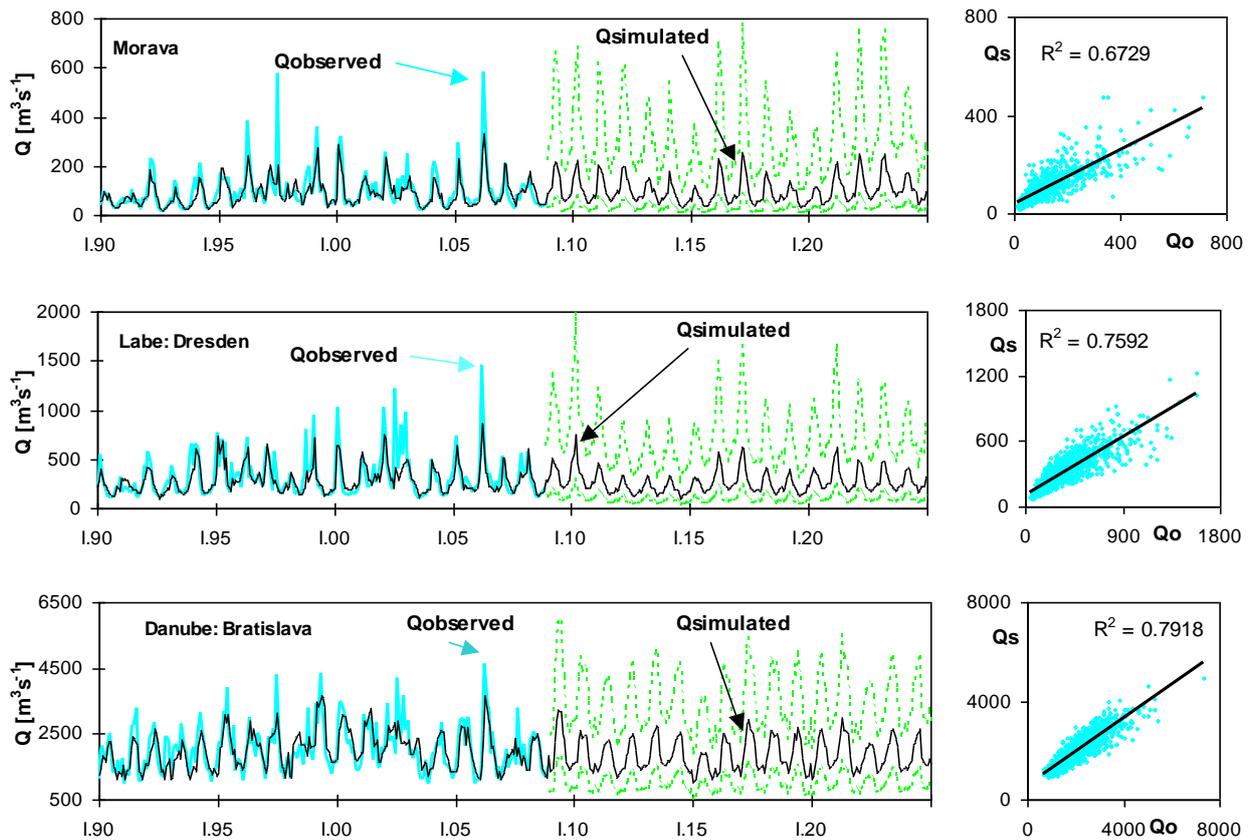
## 6 Long-term discharge prognosis

To model the discharge time series, several linear autoregressive models were tested (Komornik et al., 2006; Mares et al., 2007; Pekarova et. al, 2008).

In Figure 5 is the example of the simulated mean monthly discharge of the Danube at Bratislava 15 years ahead using models PYTHIA and SARIMA (Pekarova & Pekar, 2006).



**Figure 4.** The combined periodograms of the average annual discharges of the Morava River at M. Jan; the Danube River at Bratislava; the Elbe River at Decin and at Dresden.



**Figure 5.** Prediction of the mean monthly discharge of the Morava, Elbe, and Danube rivers, period 1990–2008 observed, period 2009–2025 simulation, model PYTHIA and SARIMA, Upper and lower 95% confidence interval. (left), observed  $Q_o$  versus simulated  $Q_s$  discharges (right).

## 7 Conclusion

The mean annual discharge of the Danube at Bratislava is almost three times higher than the Elbe discharge at Wittenberge. The difference in low flows is even bigger. Therefore the Elbe Basin is much more sensitive to water pollution and water irrigation impacts than is the Upper Danube Basin.

In the long run, the annual discharges of the Danube at the Bratislava station do not change over time. The long-term trend in the annual discharges over the period of observations is near zero.

The occurrence of the dry and wet periods is the same in both basins. These cycles are related to global atmospheric oscillations such as NAO and AO phenomena, to the Solar activity, and to the thermohaline circulation (Pekarova, 2009). The most important component of a stochastic model is the identification of the correct cycle length.

The final part of the study presented some results of the long-term prediction of average monthly discharges 15 years ahead by applying stochastic models. The results of this study suggest that in 2010 increased discharge is likely to be observed. Beyond 2012 a dry period is expected to come in the Elbe.

We are fully aware of the high uncertainty that arises from the use of such stochastic predictive models. Efficiency of predictions decreases considerably with the extent of prediction. Further analysis of several more European rivers will, however, enable to explain the long-term discharge variability which nowadays remains unknown and thus it is considered a random variable in stochastic models. Efficiency of predictions is likely to improve in the future, thus models such as these presented in this study will be widely used in hydrological practice for monthly discharge series generation, design discharge determination, etc.

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